Electromagnetics

Reference	Differential Form	Integral Form
Gauss's law	$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$
Faraday's law	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$
No magnetic charges (Gauss's law for magnetism)	$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$
Ampère's law	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right) \cdot d\mathbf{s}$

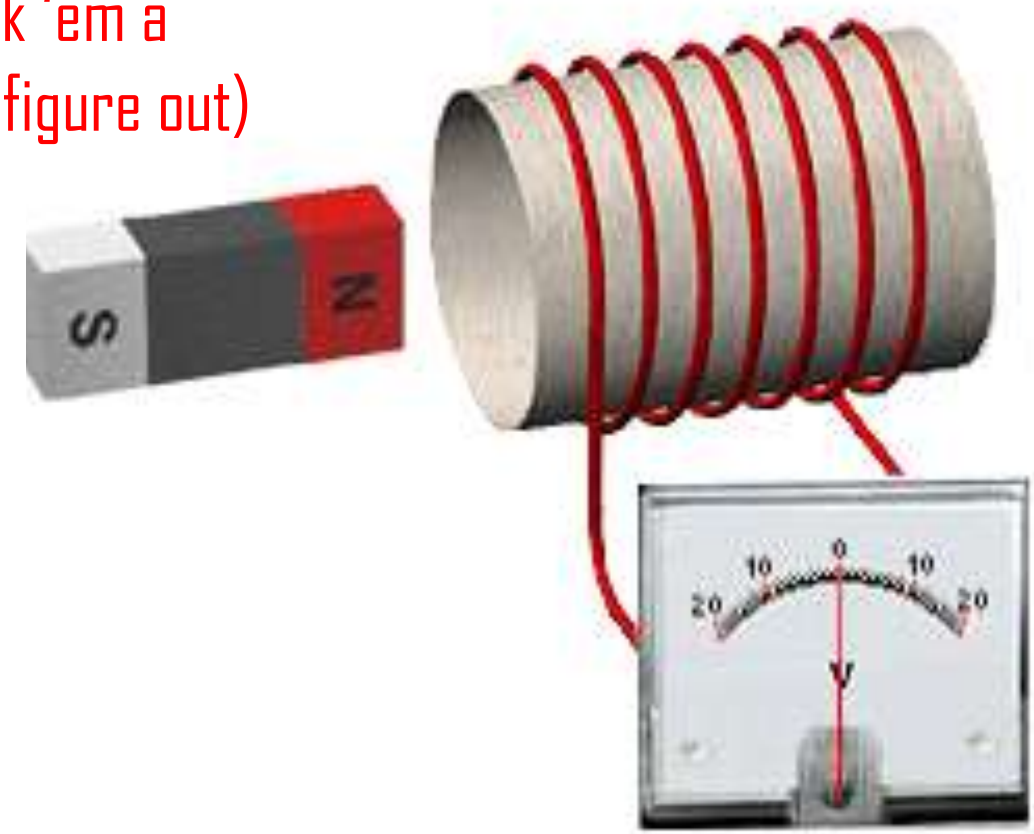
Faraday's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

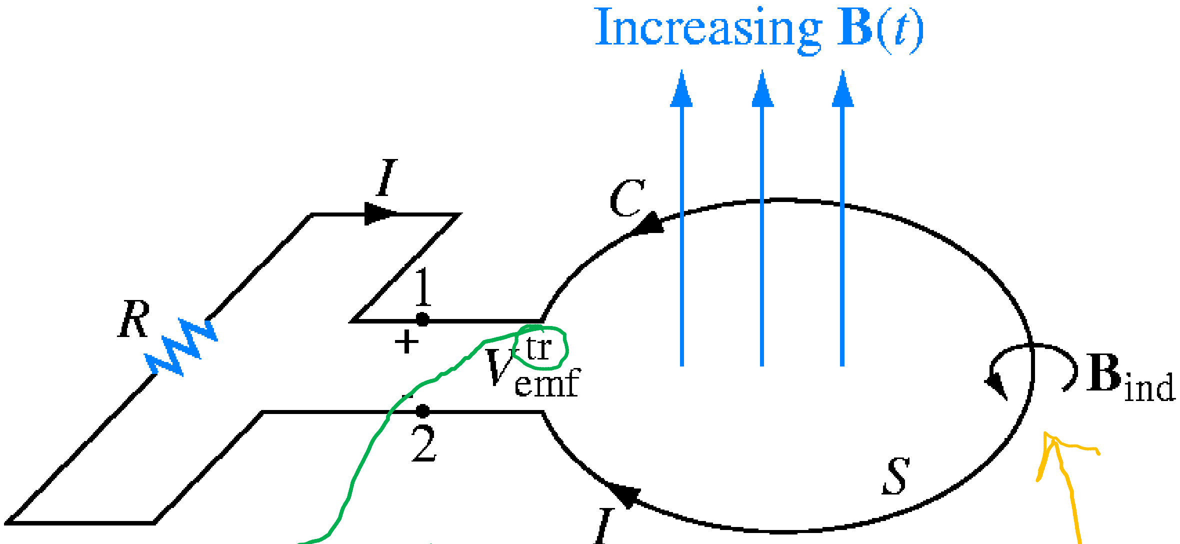
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

NOT STATIC !!

(this took 'em a while to figure out)

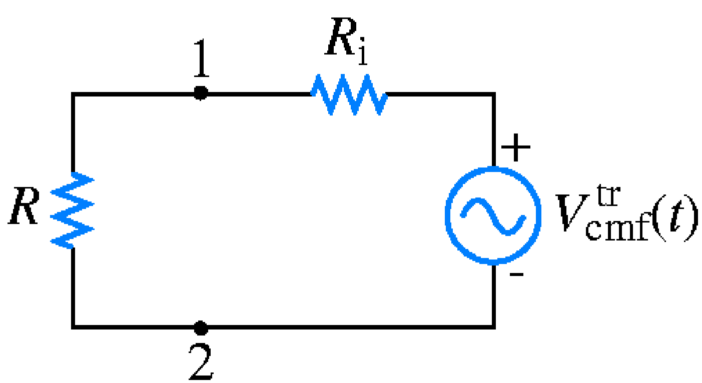


http://www.youtube.com/watch?v=stUDqGzpev8



OR motional, if $\vec{B}$ constant but loop is moving...

(a) Loop in a changing **B** field



(b) Equivalent circuit

normal component of magnetic flux

$$\Phi \equiv \int_S \vec{B} \cdot d\vec{s}$$

electromotive force

$$V_{emf} = -N \frac{\partial}{\partial t} \Phi$$

loops

change

Lenz's Law

current in loop is in direction to oppose change in magn. flux!

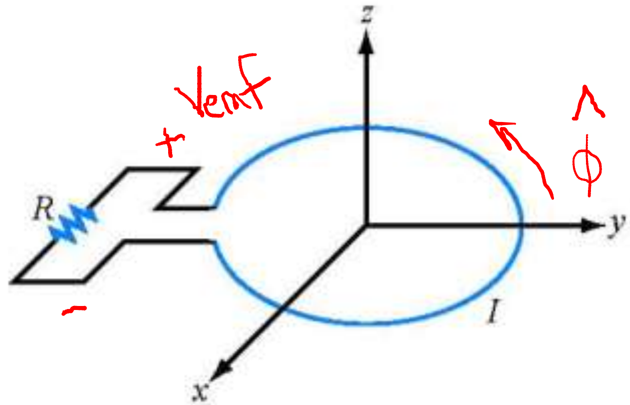
(or polarity on Vemf if I=0)

Faraday's law

anywhere, even without circuit!

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

The loop below is in the x–y plane and $\mathbf{B} = \hat{\mathbf{z}} B_0 \sin \omega t$ with B_0 positive. What is the direction of I ($\hat{\boldsymbol{\phi}}$ or $-\hat{\boldsymbol{\phi}}$) at (a) $t = 0$, (b) $\omega t = \pi/4$, and (c) $\omega t = \pi/2$?



Solution: $I = V_{\text{emf}}/R$. Since the single-turn loop is not moving or changing shape with time, $V^{\text{m}}_{\text{emf}} = 0$ V and $V_{\text{emf}} = V^{\text{tr}}_{\text{emf}}$. Therefore, from Eq. (6.8),

$$I = V^{\text{tr}}_{\text{emf}}/R = \frac{-1}{R} \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}.$$

If we take the surface normal to be $+\hat{\mathbf{z}}$, then the right hand rule gives positive flowing current to be in the $+\hat{\boldsymbol{\phi}}$ direction.

$$I = \frac{-A}{R} \frac{\partial}{\partial t} B_0 \sin \omega t = \frac{-A B_0 \omega}{R} \cos \omega t \quad \text{(A)},$$

where A is the area of the loop.

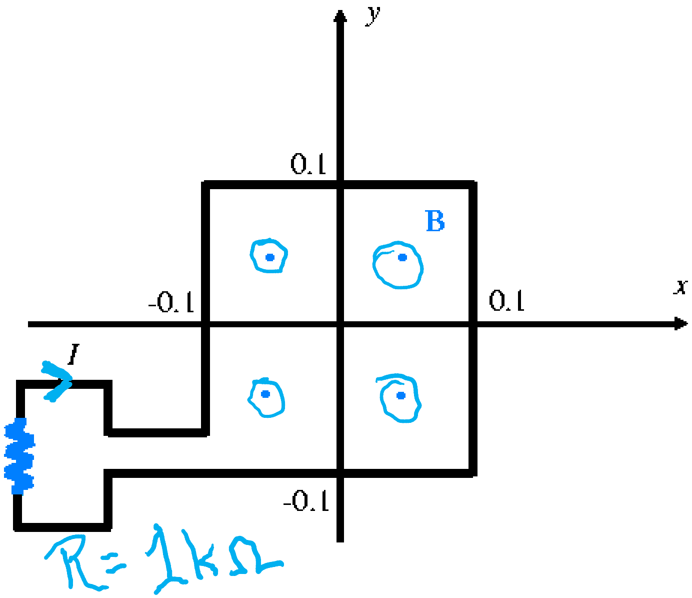
(a) A, ω and R are positive quantities. At $t = 0$, $\cos \omega t = 1$ so $I < 0$ and the current is flowing in the $-\hat{\boldsymbol{\phi}}$ direction (so as to produce an induced magnetic field that opposes **B**).

(b) At $\omega t = \pi/4$, $\cos \omega t = \sqrt{2}/2$ so $I < 0$ and the current is still flowing in the $-\hat{\boldsymbol{\phi}}$ direction.

(c) At $\omega t = \pi/2$, $\cos \omega t = 0$ so $I = 0$. There is no current flowing in either direction.

Q: what is Vemf if R→∞ (i.e. open ckt.)?

Consider a 10-turn square loop centered at the origin and having 20-cm sides oriented parallel to the x- and y-axes. If $\mathbf{B} = \hat{\mathbf{z}} B_0 x^2 \cos 10^3 t$ and $B_0 = 100$ T, find the current in the circuit.



$$
\begin{aligned}
\Phi &= \int_S \mathbf{B} \cdot d\mathbf{s} \\
&= \int_{x=-0.1}^{0.1} \int_{y=-0.1}^{0.1} (\hat{\mathbf{z}}\, 100x^2 \cos 10^3 t) \cdot \hat{\mathbf{z}}\, dx\, dy \\
&= (100 \cos 10^3 t) \times 0.2 \int_{-0.1}^{0.1} x^2\, dx \\
&= 20 \cos 10^3 t \left. \frac{x^3}{3} \right|_{-0.1}^{0.1} \\
&= \frac{20}{3} \cos 10^3 t ((0.1)^3 + (0.1)^3) = 13.3 \times 10^{-3} \cos 10^3 t.
\end{aligned}
$$

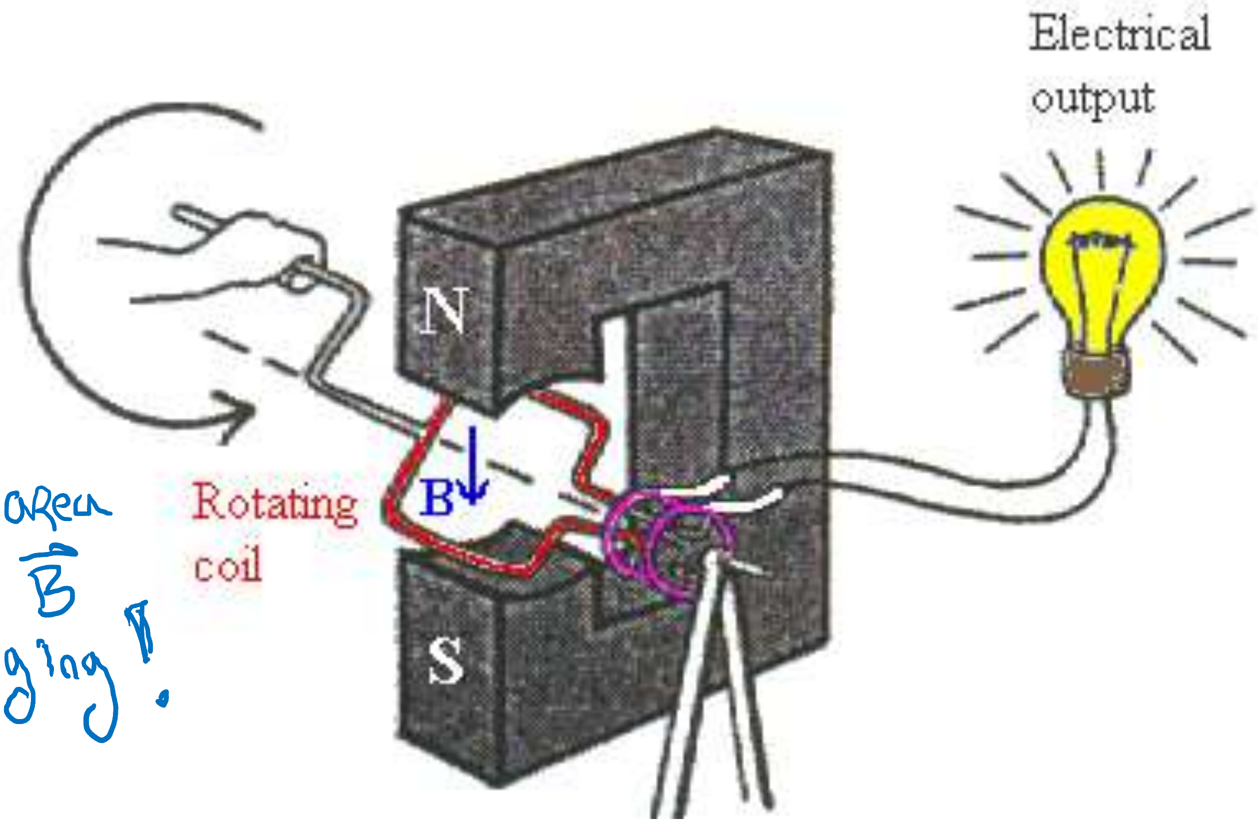
$$
\begin{aligned}
\cancel{I} &= \frac{V_{\text{emf}}}{R} \\
&= -\frac{N}{R} \frac{d\Phi}{dt} \\
&= -\frac{10}{1000} \frac{d}{dt} (13.3 \times 10^{-3} \cos 10^3 t) = 133 \sin 10^3 t \quad \text{(mA)}.
\end{aligned}
$$

Q: what if $\vec{B} = \hat{x} \sin 10^3 t$??

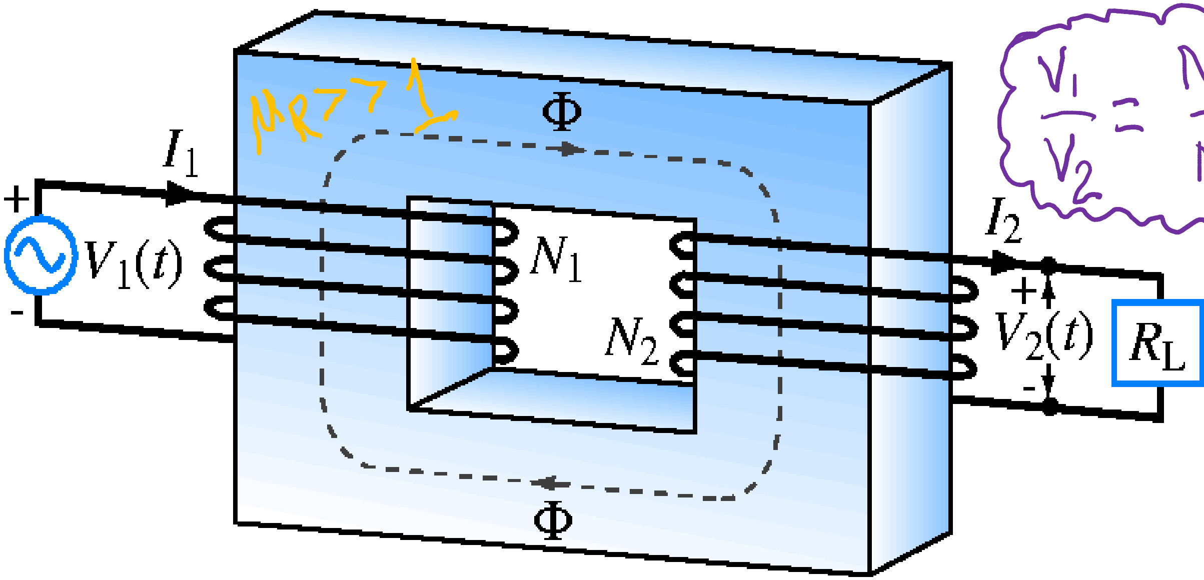
At $t = 0$, $d\Phi/dt < 0$ and $V_{\text{emf}} > 0$. Since the flux is decreasing, Lenz's law requires I to be in the direction opposite that shown in the figure so that the flux inducted by I is in opposition to the trend of $d\Phi/dt$. Hence, in terms of the indicated direction of I,

$$
I = -133 \sin 10^3 t \quad \text{(mA)}.
$$

$\vec{B}$ constant, but loop area normal to $\vec{B}$ is changing!





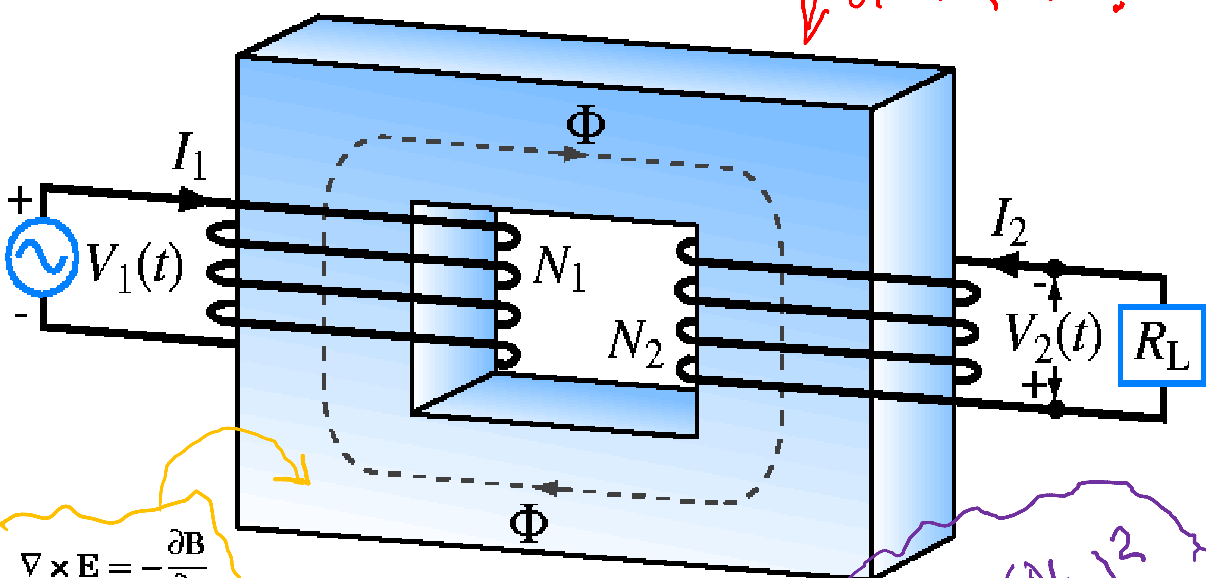


$\mu_R >> 1$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

(a)

what's different?

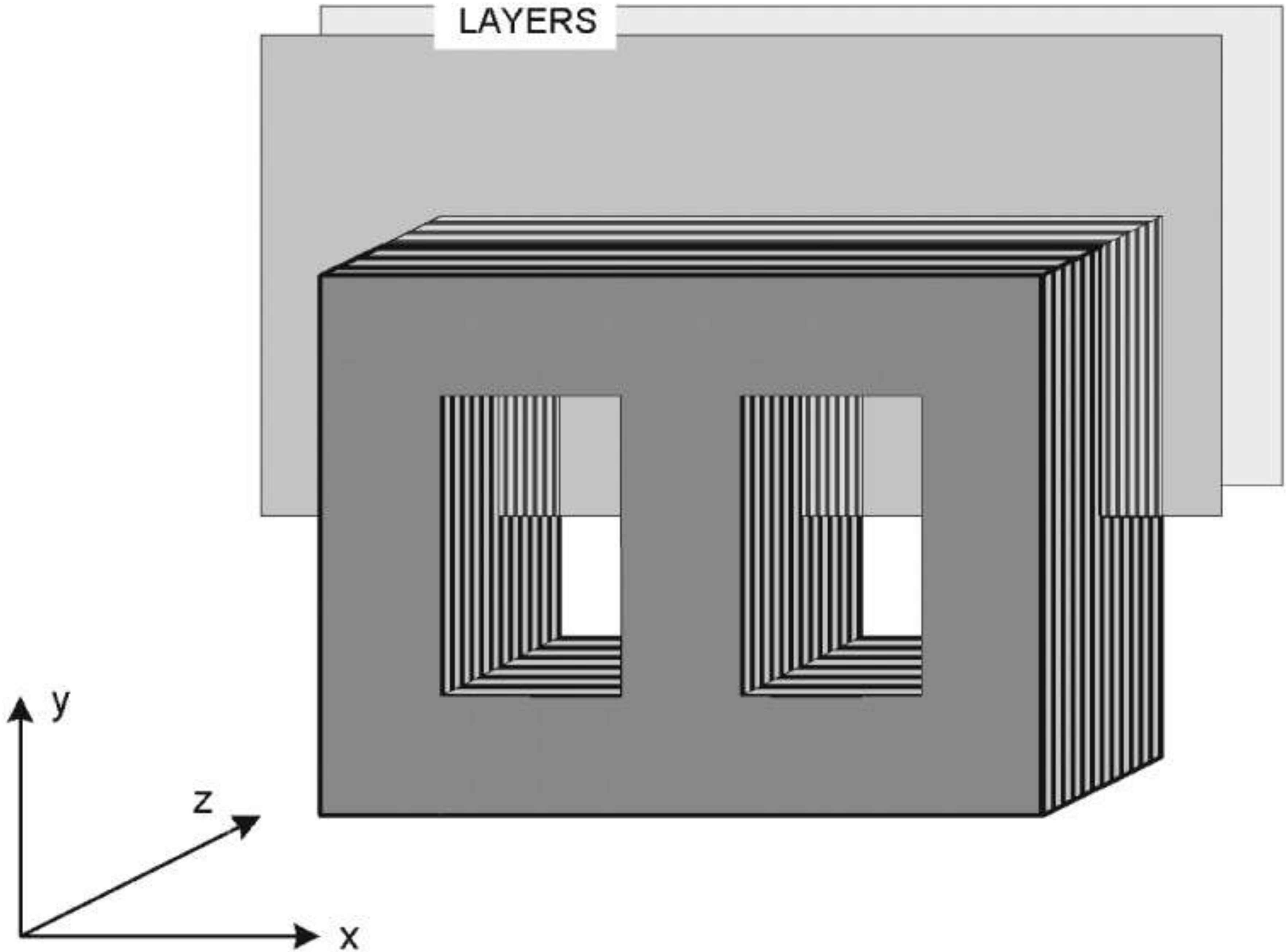


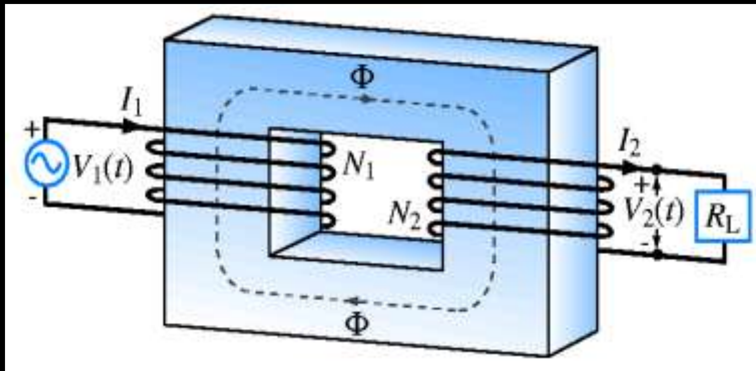
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

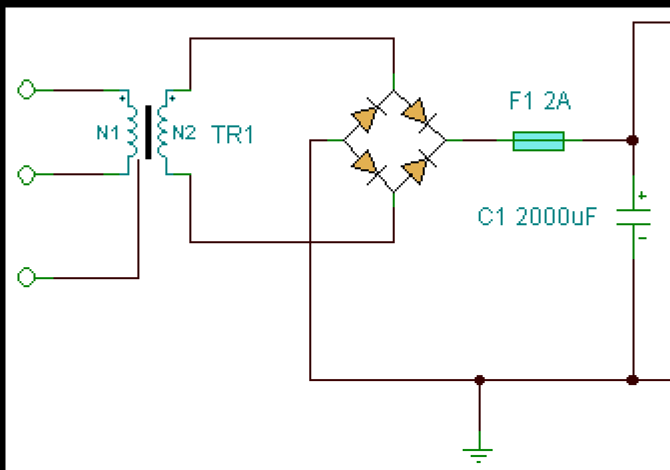
$$J = \sigma E$$

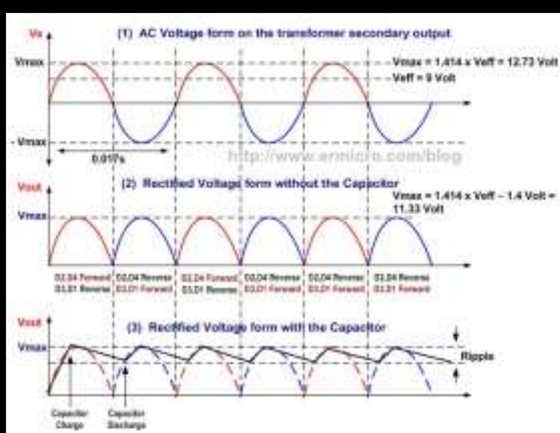
$$R_{in} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

 (b)

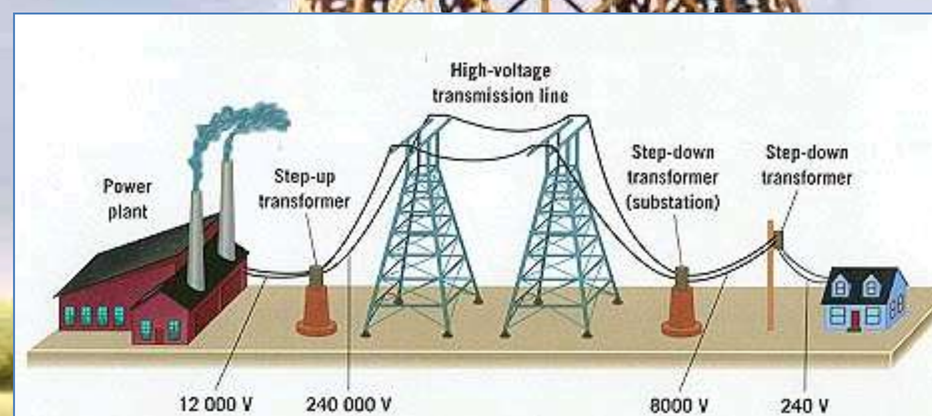

LAYERS
y
z
x

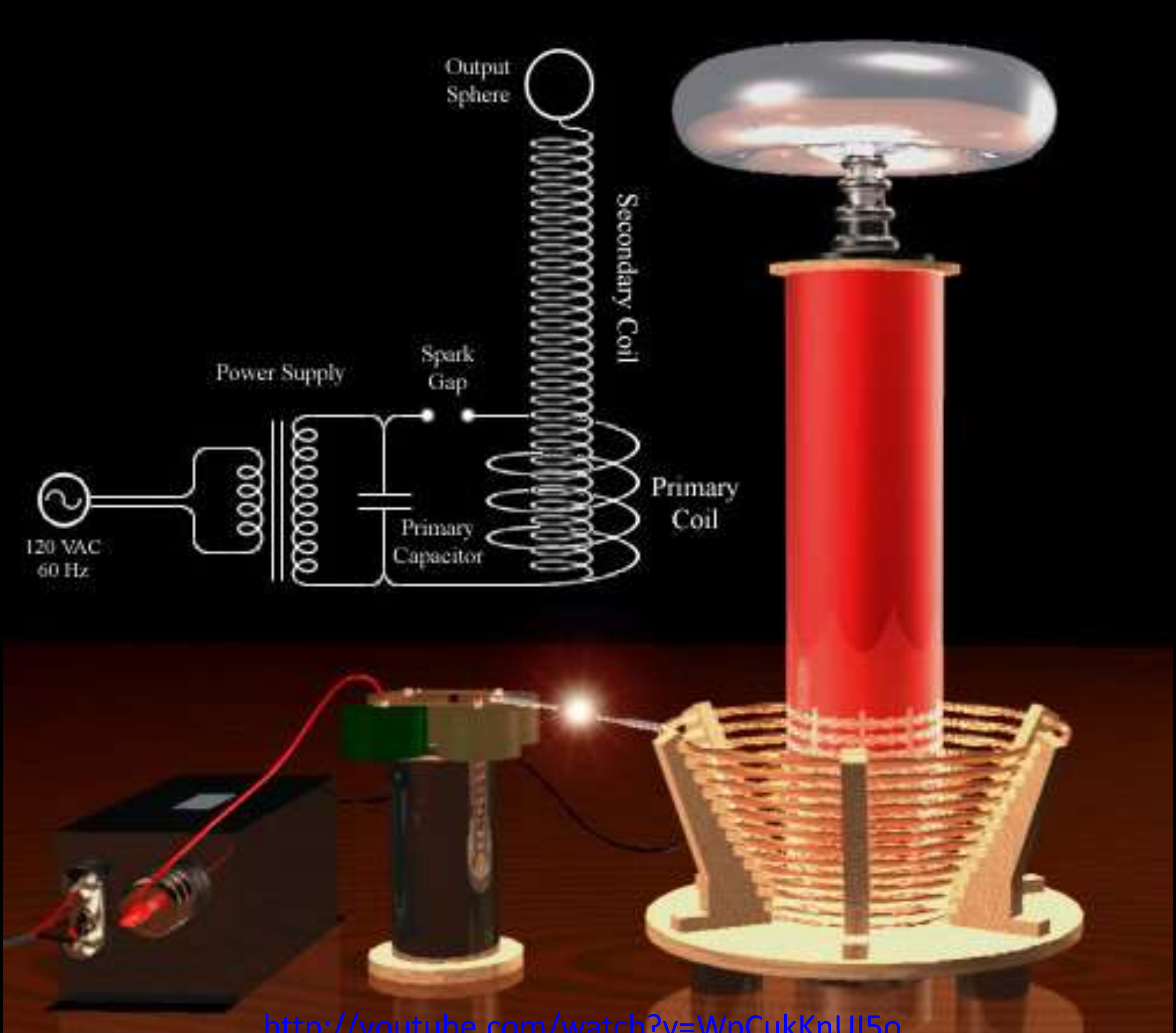






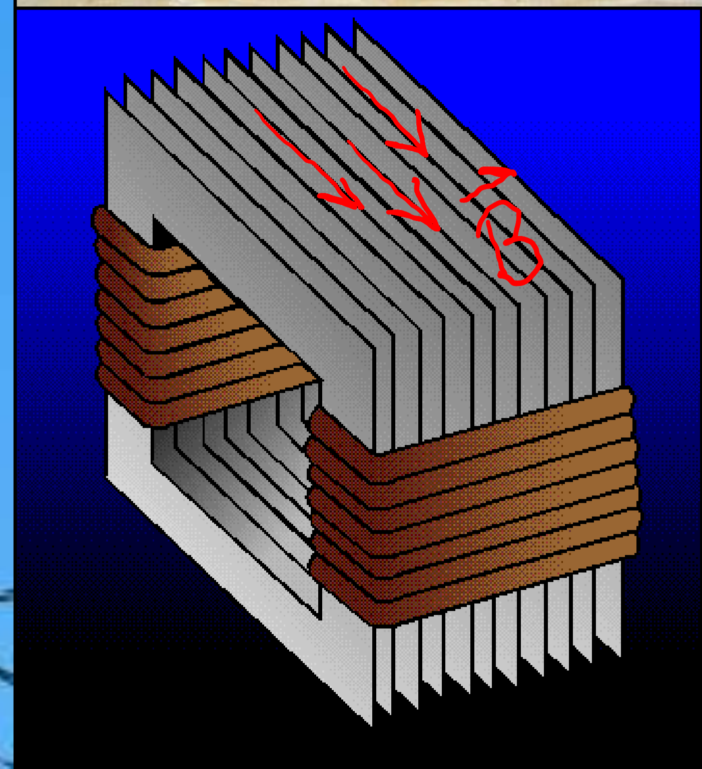
The AC to DC full wave rectifier Voltage time diagram



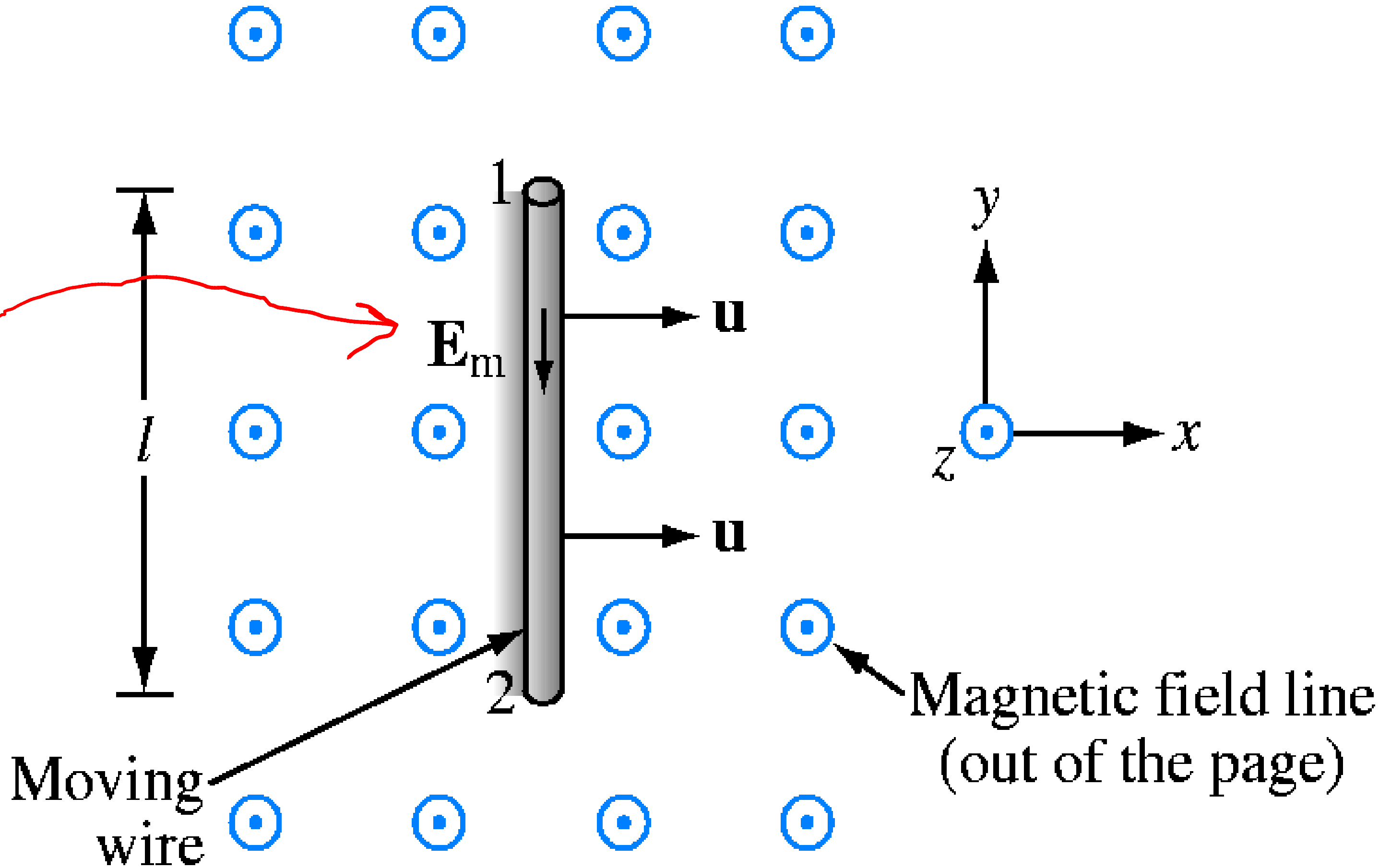


http://youtube.com/watch?v=WpCukKnUI5o

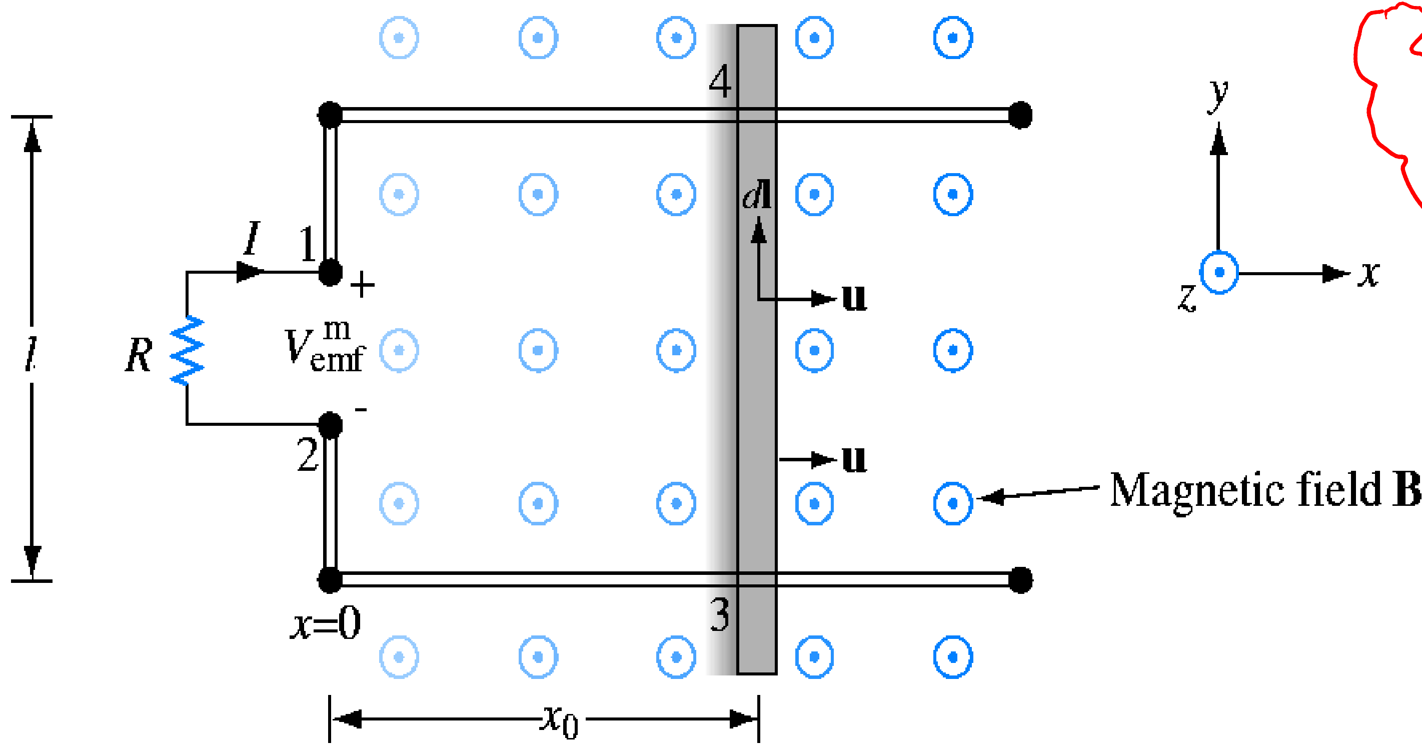

SHELL
Motor
OIL
Motor
SHELL
Tractor
OIL
SHELL
Aircraft
OIL



Motional Electric Field

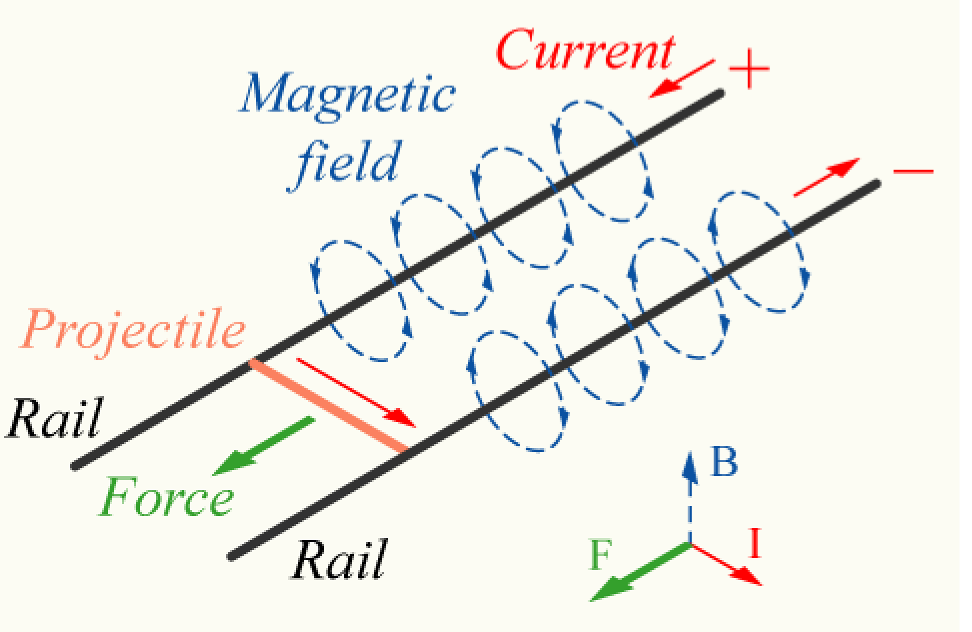


http://www.tethers.com/TethersGeneral.html



Sliding Bar

http://www.youtube.com/watch?v=y54aLcC3G74



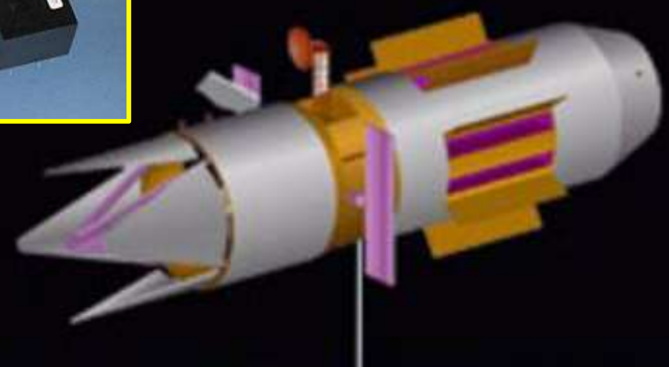
If you could design the power grid from the ground up, what would you do??

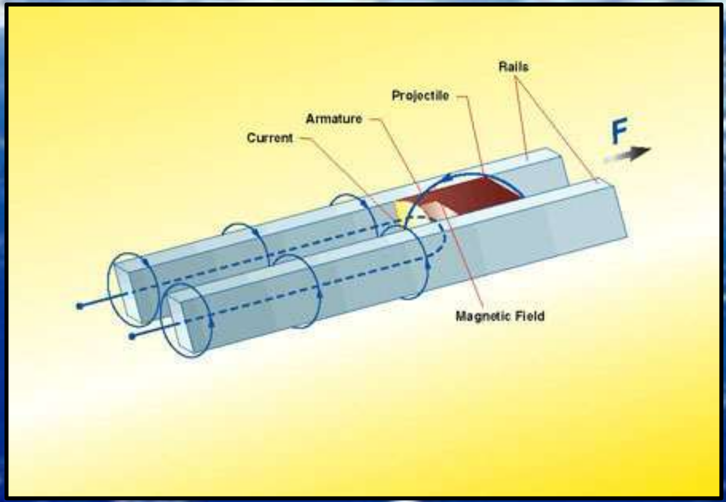
Probably go dc for transmission and distribution:
1.dc transmission lines have higher power density and no stability limits.
2.power converters are smaller and more flexible than transformers.
3.simplifies interface to emerging energy sources and storage technologies (e.g., photovoltaics, fuel cells, flywheels, superconducting energy storage) and increasingly common electronic loads (e.g., computers/servers and motor drives).
Generation using conventional energy sources (e.g., coal, nuclear) would still be ac, rectifiers would be used to interface to dc transmission.

It's a good question. You might want to increase the frequency, as it would reduce the size (and hence cost) of the transformers. Or you might want to go completely DC, using high-power DC-DC converters instead of transformers for voltage conversion. A DC system would be more controllable and hence more stable than AC systems.

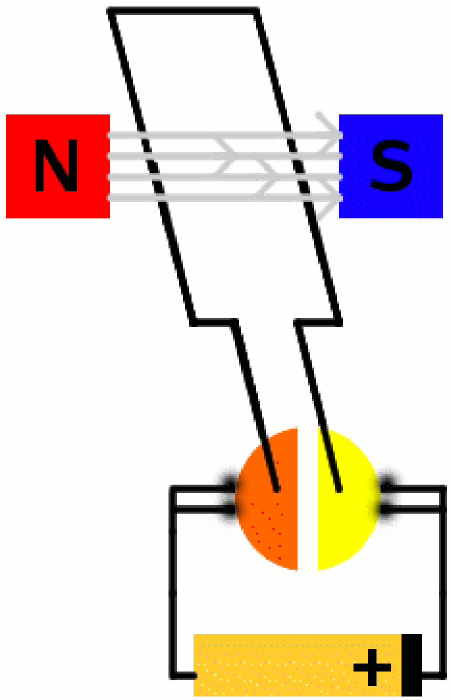
ELECTROMAGNETIC INDUCTION

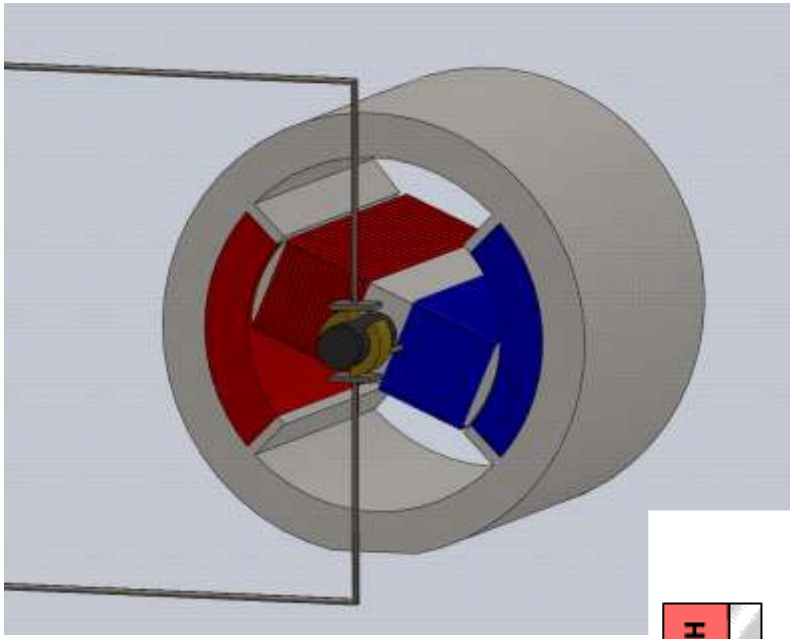
$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{s}$$



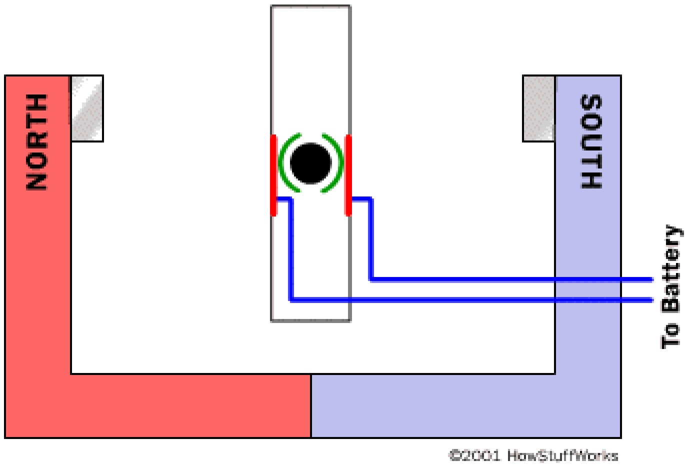


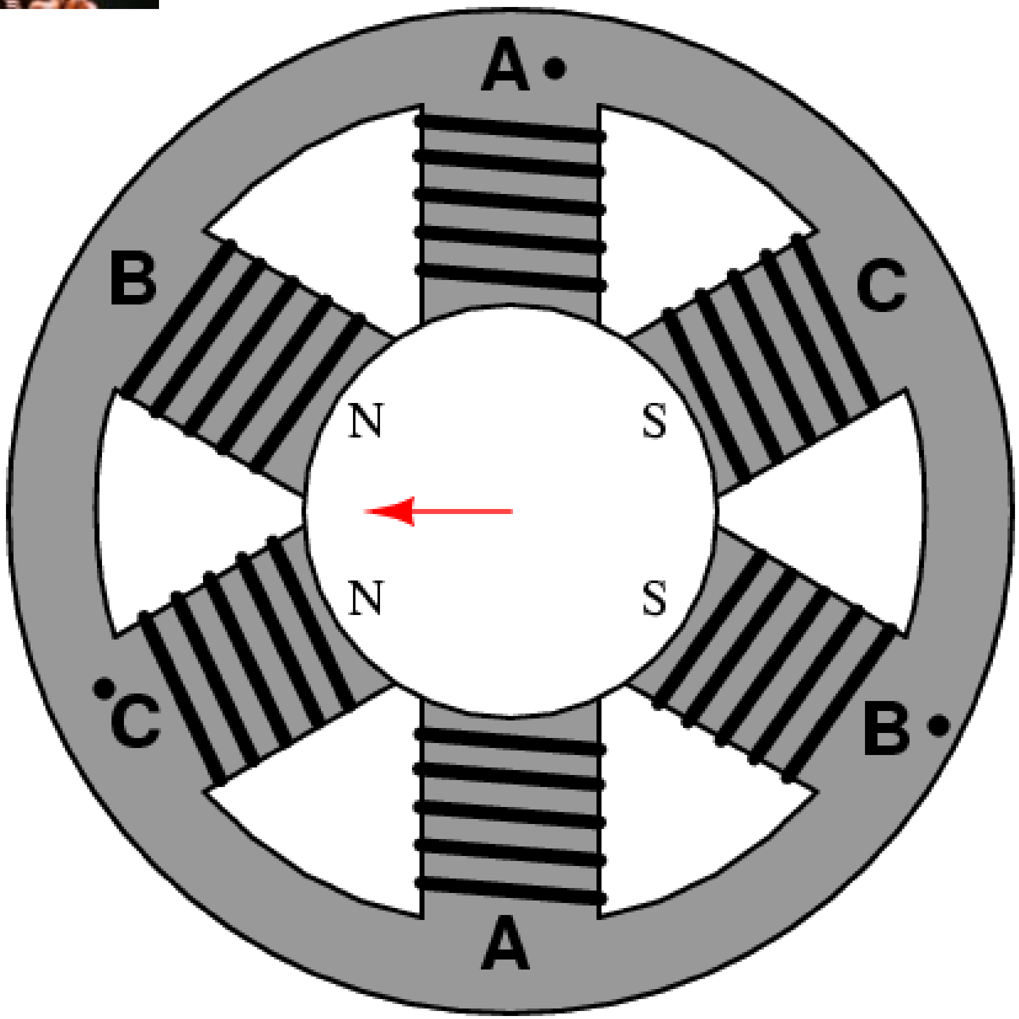
Lecture 25

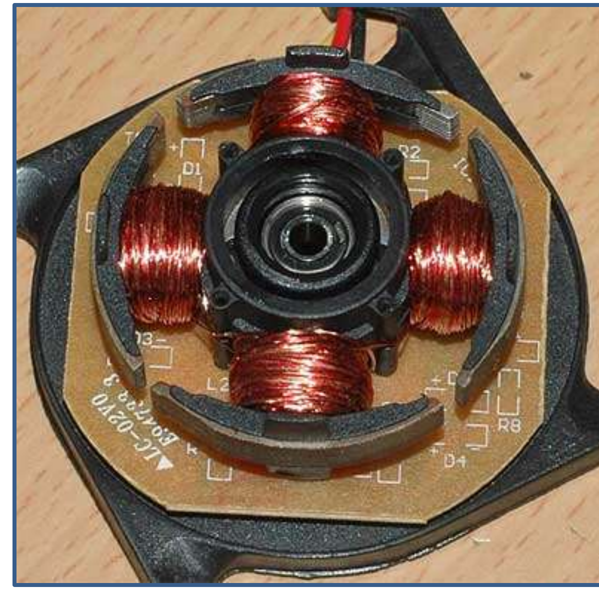


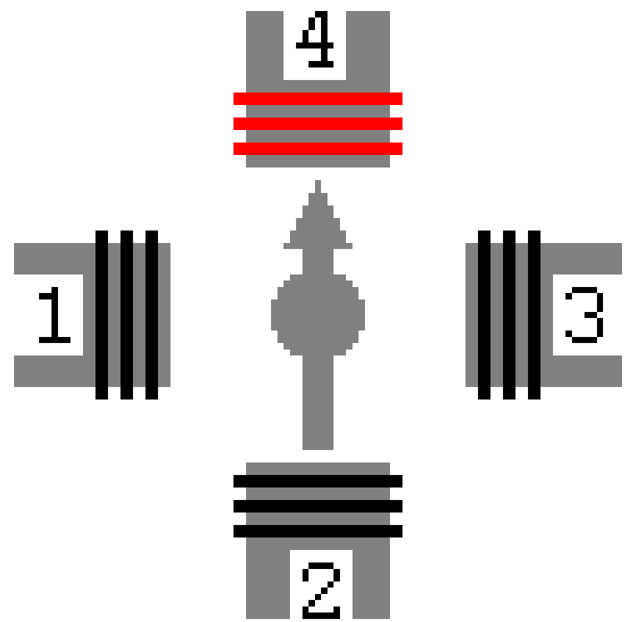


an aside









http://users.tinyworld.co.uk/flecc/brushless_motor.html

ELECTROMAGNETIC INDUCTION

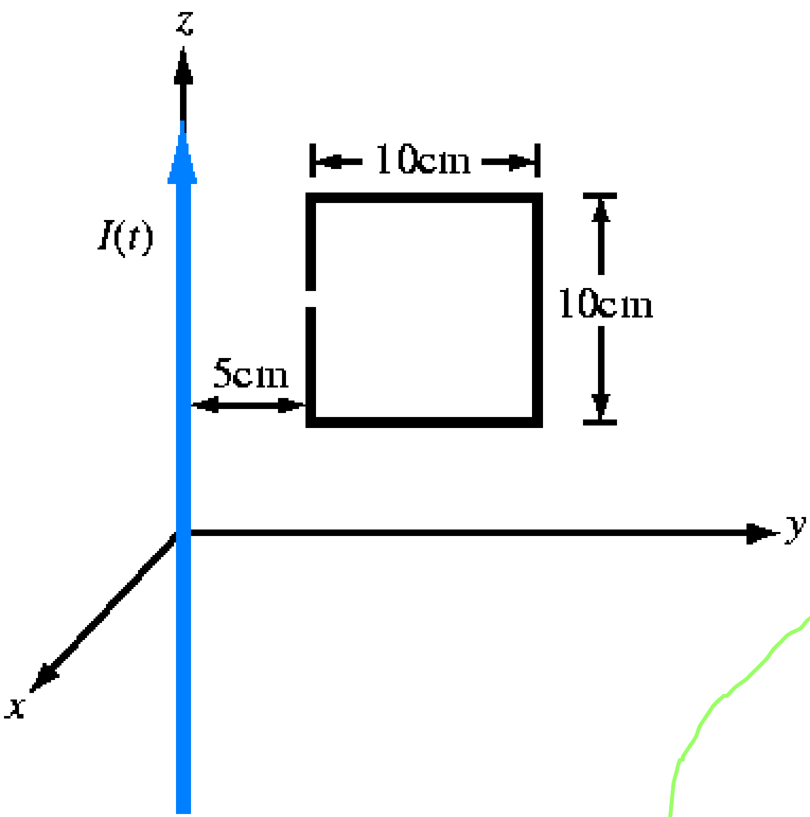
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

The loop shown below is coplanar with a long, straight wire carrying a current

$$I(t) = 5\cos 2\pi \times 10^4 t \quad \text{(A)}.$$

(a) Determine the emf induced across a small gap created in the loop.

(b) Determine the direction and magnitude of the current that would flow through a 4-Ω resistor connected across the gap. The loop has an internal resistance of 1 Ω.



Solution:

(a) The magnetic field due to the wire is

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$ and $r = y$. The flux passing through the loop is

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_{5\text{ cm}}^{15\text{ cm}} \left(-\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y}\right) \cdot [-\hat{\mathbf{x}}\, 10\text{ (cm)}]\, dy$$
$$= \frac{\mu_0 I \times 10^{-1}}{2\pi} \ln \frac{15}{5}$$
$$= \frac{4\pi \times 10^{-7} \times 5\cos(2\pi \times 10^4 t) \times 10^{-1}}{2\pi} \times 1.1$$
$$= 1.1 \times 10^{-7} \cos(2\pi \times 10^4 t) \quad \text{(Wb)}.$$

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = 1.1 \times 2\pi \times 10^4 \sin(2\pi \times 10^4 t) \times 10^{-7}$$
$$= 6.9 \times 10^{-3} \sin(2\pi \times 10^4 t) \quad \text{(V)}.$$

(b)

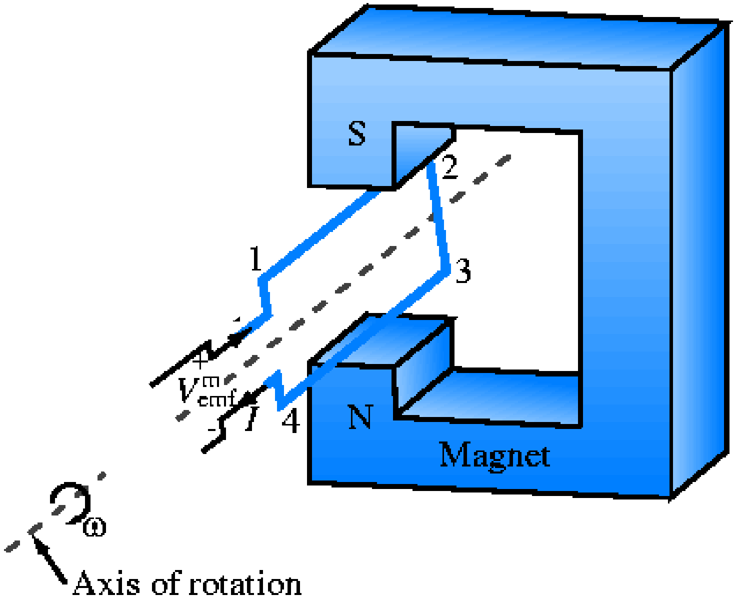
$$I_{\text{ind}} = \frac{V_{\text{emf}}}{4+1} = \frac{6.9 \times 10^{-3}}{5} \sin(2\pi \times 10^4 t) = 1.38 \sin(2\pi \times 10^4 t) \quad \text{(mA)}.$$

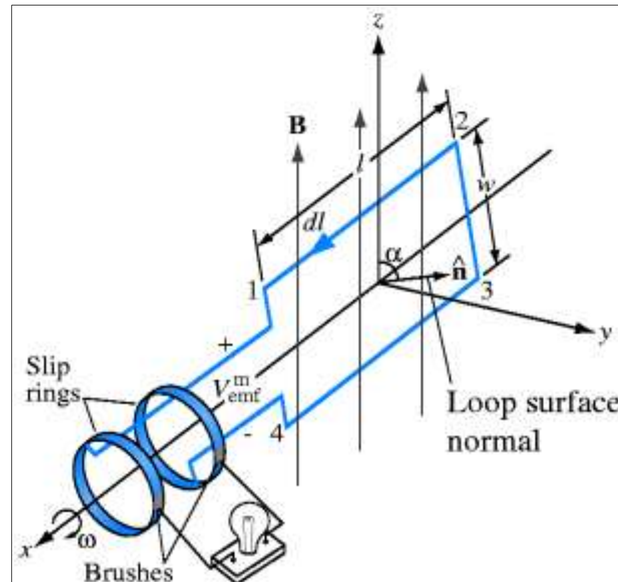
At $t = 0$, **B** is a maximum, it points in $-\hat{\mathbf{x}}$-direction, and since it varies as $\cos(2\pi \times 10^4 t)$, it is decreasing. Hence, the induced current has to be ~~C~~CW when looking down on the loop, as shown in the figure.

Q: what if there are 2 gaps?

A rectangular conducting loop 5 cm $\times$10 cm with a small air gap in one of its sides is spinning at 7200 revolutions per minute. If the field **B** is normal to the loop axis and its magnitude is 6×10^{-6} T, what is the peak voltage induced across the air gap?

is this arbitrary?





Solution:

gotta work in ω

$$\omega = \frac{2\pi \text{ rad/cycle} \times 7200 \text{ cycles/min}}{60 \text{ s/min}} = 240\pi \text{ rad/s},$$

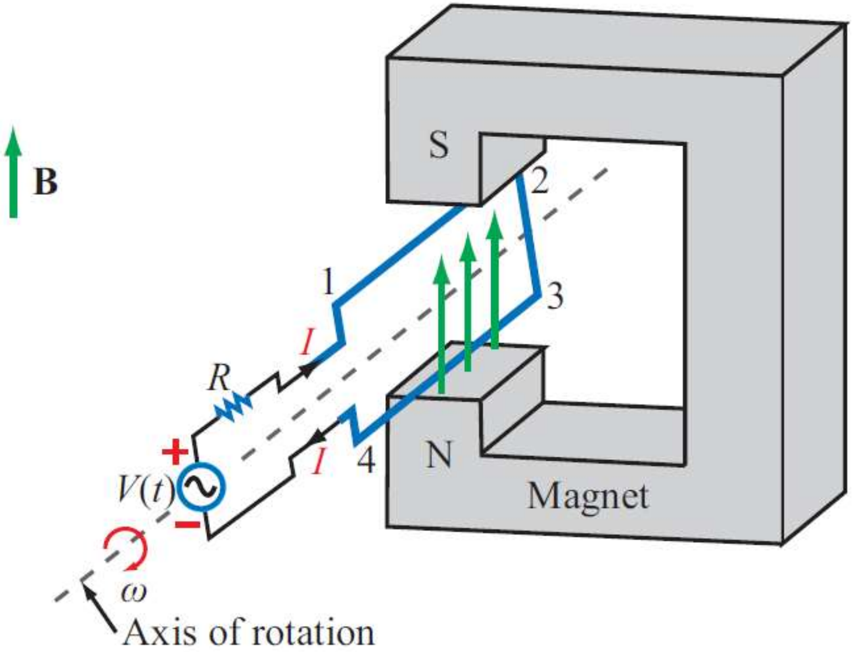
$$A = 5 \text{ cm} \times 10 \text{ cm}/(100 \text{ cm/m})^2 = 5.0 \times 10^{-3} \text{ m}^2.$$

From Eqs. (6.36) or (6.38), $V_{\text{emf}} = A\omega B_0 \sin\omega t$; it can be seen that the peak voltage is

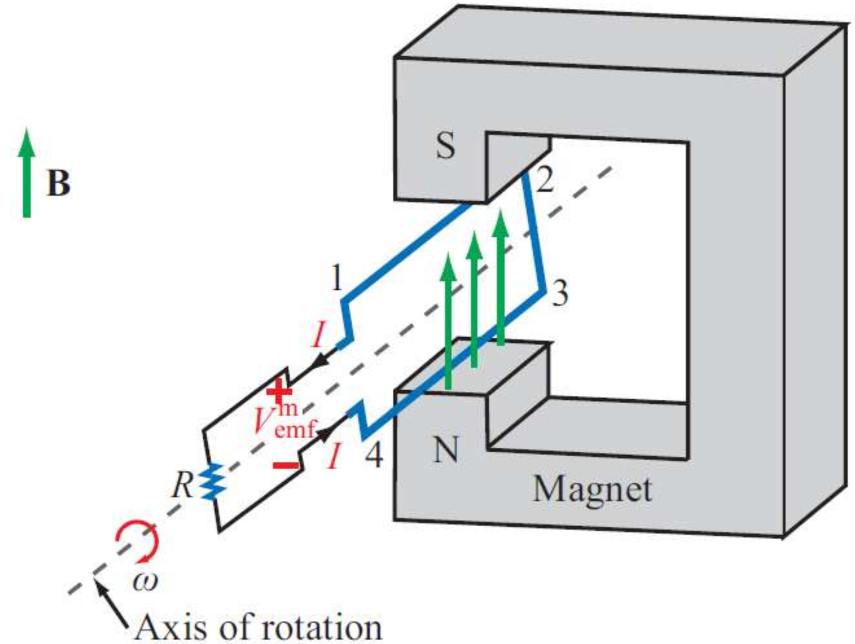
$$V_{\text{emf}}^{\text{peak}} = A\omega B_0 = 5.0 \times 10^{-3} \times 240\pi \times 6 \times 10^{-6} = 22.62 \quad (\mu\text{V}).$$

Q: what is freq. of $V_{emf}(t)$?

EM Motor/ Generator Reciprocity



(a) ac motor



(b) ac generator

Motor: Electrical to mechanical energy conversion

Generator: Mechanical to electrical energy conversion

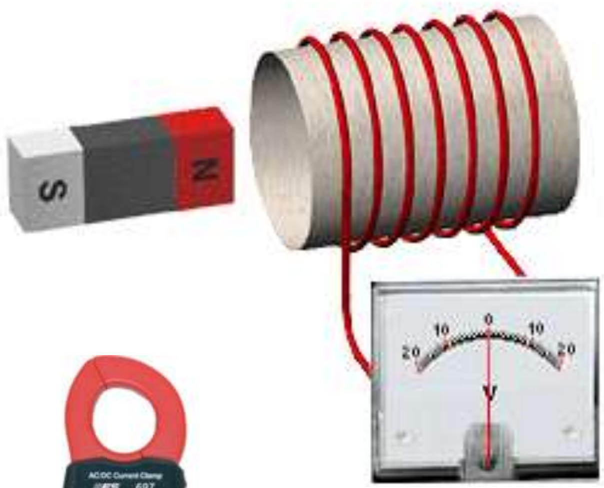

EVEREADY

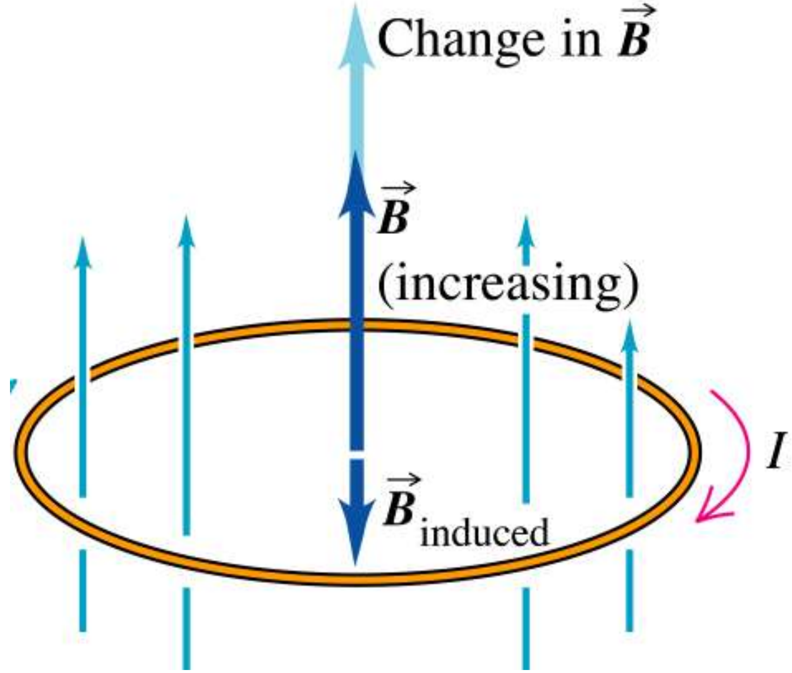
Dynamic Fields (continued)

6-7 thru 6-10
(and look at 6-11)

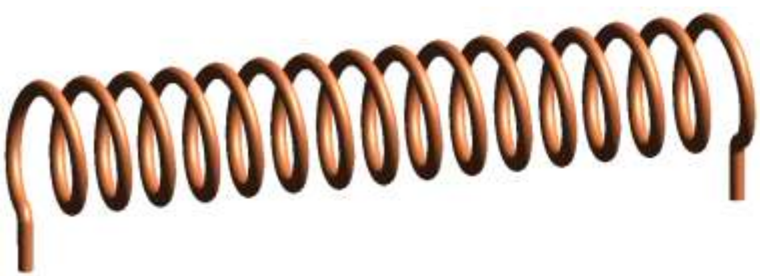
… time-varying magnetic fields.

Faraday's law $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ $\qquad \oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \dfrac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$





$$V = L\frac{dI}{dt}$$

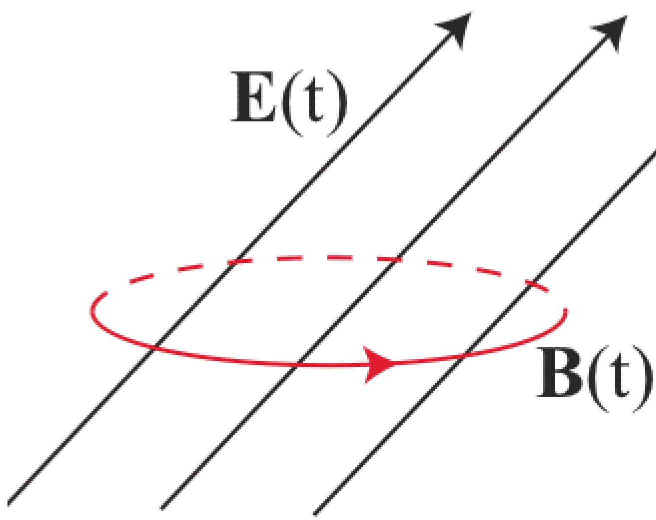


…but what about time-varying electric fields?

Ampère's law

http://www.youtube.com/watch?v=lAWZJrG6TX8

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \qquad \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}$$



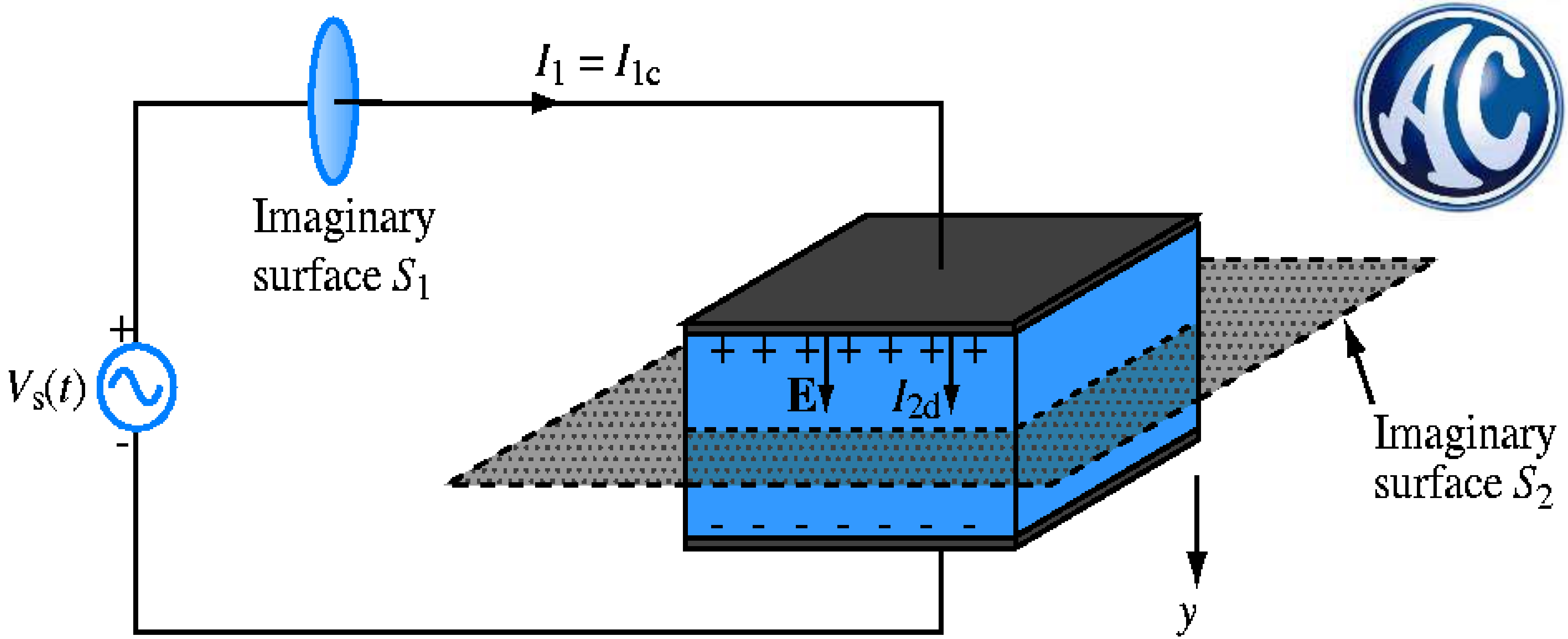
$$= \underbrace{\int_S \vec{J} \cdot d\vec{s}}_{I = \text{conduction current}} + \underbrace{\int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}}_{\text{Displacement Current}}$$

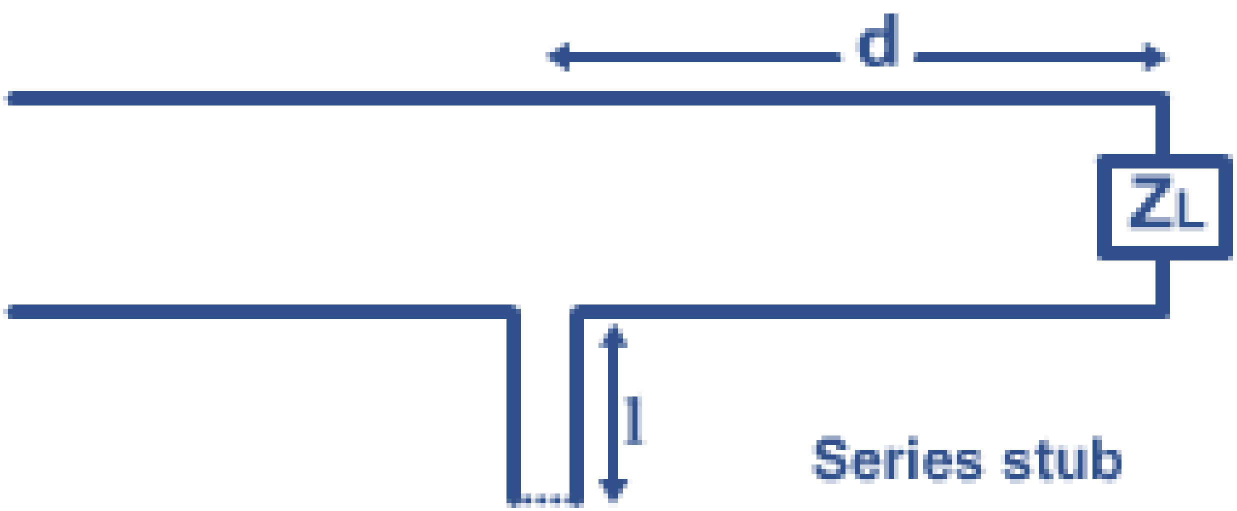
- not actually moving charge, but behaves Like current.
- Maxwell's addition

Q: why doesn't

$$\nabla \times \mathbf{E} = ? \; -\frac{\partial \mathbf{B}}{\partial t}$$

?? ?


$I_1 = I_{1c}$
Imaginary surface S_1
$V_s(t)$
+
-
E
I_{2d}
Imaginary surface S_2
y


d
ZL
l
Series stub

The plates of a parallel-plate capacitor have areas 10 cm^2 each and are separated by 2 cm. The capacitor is filled with a dielectric material with $\varepsilon = 4\varepsilon_0$, and the voltage across it is given by $V(t) = 30 \cos 2\pi \times 10^6 t$ (V). Find the displacement current.

$$C = \frac{\epsilon A}{d} = \frac{Q}{V}$$

Solution: Since the voltage is of the form given by Eq. (6.46) with $V_0 = 30$ V and $\omega = 2\pi \times 10^6$ rad/s, the displacement current is given by Eq. (6.49):

$$I_d = -\frac{\varepsilon A}{d} V_0 \omega \sin \omega t = \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$= -\frac{4 \times 8.854 \times 10^{-12} \times 10 \times 10^{-4}}{2 \times 10^{-2}} \times 30 \times 2\pi \times 10^6 \sin(2\pi \times 10^6 t)$$

$$= -0.33 \sin(2\pi \times 10^6 t) \quad \text{(mA)}.$$

Don't just plug & chug!!!

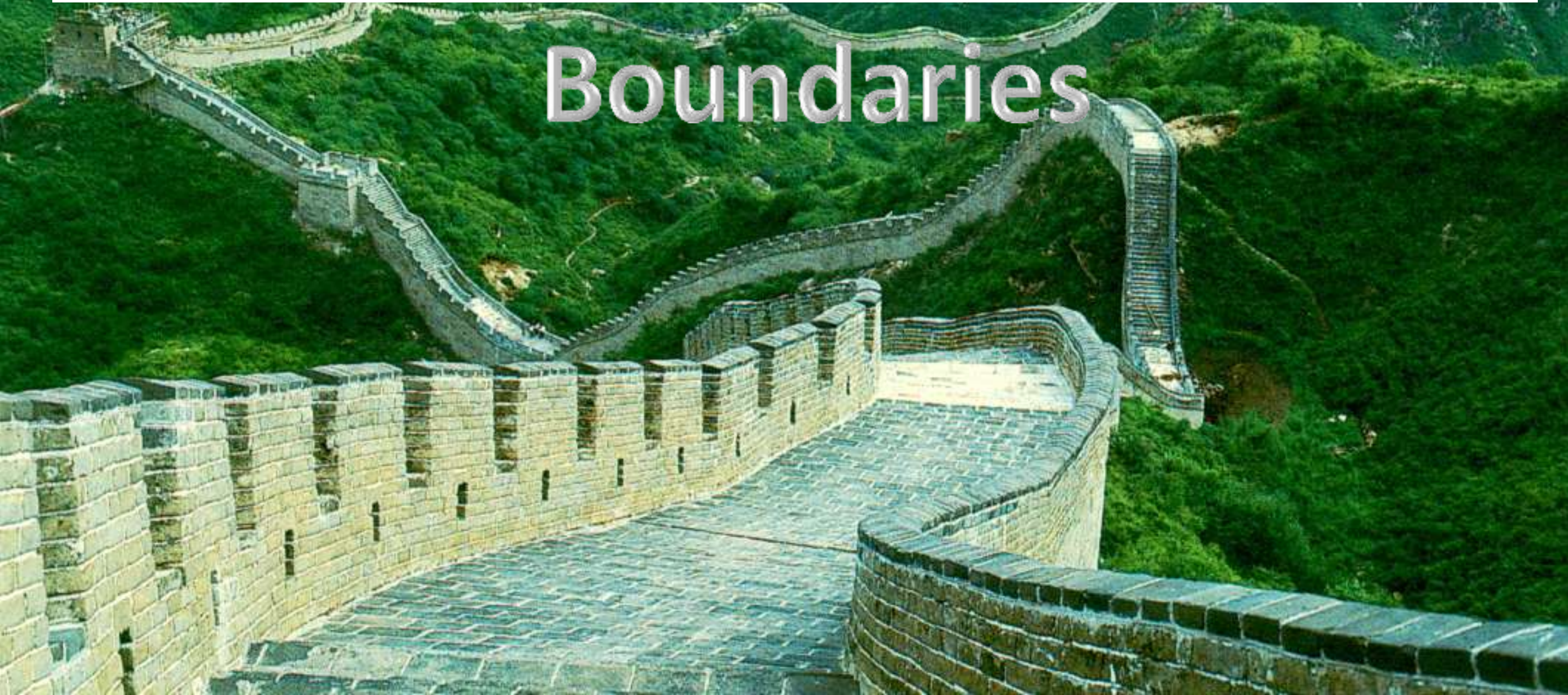
$$= C \frac{dV}{dt}$$

Yea 210!

Field Components	General Form	Medium 1 Dielectric	Medium 2 Dielectric	Medium 1 Dielectric	Medium 2 Conductor
Tangential E	$\hat{\mathbf{n}}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$	$E_{1t} = E_{2t}$		$E_{1t} = E_{2t} = 0$	
Normal D	$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$		$D_{1n} = \rho_s$	$D_{2n} = 0$
Tangential H	$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$	$H_{1t} = H_{2t}$		$H_{1t} = J_s$	$H_{2t} = 0$
Normal B	$\hat{\mathbf{n}}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$	$B_{1n} = B_{2n}$		$B_{1n} = B_{2n} = 0$	

Notes: (1) ρ_s is the surface charge density at the boundary; (2) $\mathbf{J}_s$ is the surface current density at the boundary; (3) normal components of all fields are along $\hat{\mathbf{n}}_2$, the outward unit vector of medium 2; (4) $E_{1t} = E_{2t}$ implies that the tangential components are equal in magnitude and parallel in direction; (5) direction of $\mathbf{J}_s$ is orthogonal to $(\mathbf{H}_1 - \mathbf{H}_2)$.

Boundaries



$$I = dQ/dt$$

$$\oint_S \vec{J} \cdot d\vec{S} = -\frac{d}{dt}\int_V \rho_v \, dv$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

In conductor

$$\vec{J} = \sigma \vec{E} \rightarrow \sigma \underbrace{\nabla \cdot \vec{E}}_{\rho_v/\epsilon} = -\frac{\partial \rho_v}{\partial t} \rightarrow \rho_v = \rho_{v_0} e^{-t/\tau_r}$$

with

$$\tau = \epsilon/\sigma$$

Relaxation time constant

Charge Current Continuity Equation

Current I out of a volume is equal to rate of decrease of charge Q contained in that volume:

$$I = -\frac{dQ}{dt} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_{\mathrm{v}}\, d\mathcal{V}$$

$$\oint_S \mathbf{J}\cdot d\mathbf{s} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_{\mathrm{v}}\, d\mathcal{V}$$

$$\oint_S \mathbf{J}\cdot d\mathbf{s} = \int_{\mathcal{V}} \nabla\cdot\mathbf{J}\, d\mathcal{V} = -\frac{d}{dt}\int_{\mathcal{V}} \rho_{\mathrm{v}}\, d\mathcal{V}$$

Used Divergence Theorem

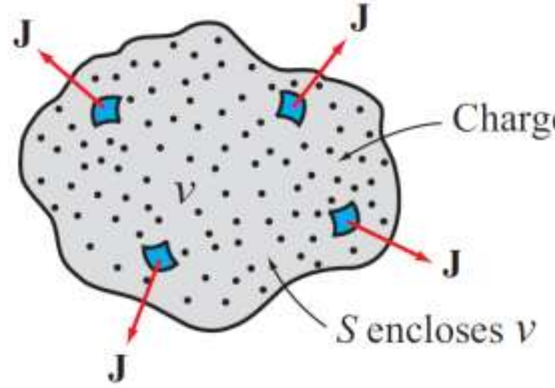


Figure 6-14: The total current flowing out of a volume $\mathcal{V}$ is equal to the flux of the current density **J** through the surface S, which in turn is equal to the rate of decrease of the charge enclosed in $\mathcal{V}$.

$$\boxed{\nabla\cdot\mathbf{J} = -\frac{\partial\rho_{\mathrm{v}}}{\partial t}, \qquad (6.54)}$$

which is known as the ***charge-current continuity relation***, or simply the ***charge continuity equation***.

At $t = 0$, charge density ρ_{v0} was introduced into the interior of a material with a relative permittivity $\varepsilon_r = 9$. If at $t = 1$ μs the charge density has dissipated down to $10^{-3}\rho_{v0}$, what is the conductivity of the material?

is this big, small or what?

Solution: We start by using Eq. (6.61) to find τ_r:

$$\rho_v(t) = \rho_{v0} e^{-t/\tau_r},$$

or

$$10^{-3}\rho_{v0} = \rho_{v0} e^{-10^{-6}/\tau_r},$$

which gives

Can have <u>both</u> ε and σ ☺

$$\ln 10^{-3} = -\frac{10^{-6}}{\tau_r},$$

or

$$\tau_r = -\frac{10^{-6}}{\ln 10^{-3}} = 1.45 \times 10^{-7} \quad \text{(s)}.$$

Is this a good conductor?

But $\tau_r = \varepsilon/\sigma = 9\varepsilon_0/\sigma$. Hence

$$\sigma = \frac{9\varepsilon_0}{\tau_r} = \frac{9 \times 8.854 \times 10^{-12}}{1.45 \times 10^{-7}} = 5.5 \times 10^{-4} \quad \text{(S/m)}.$$

Time Harmonic Potentials

If charges and currents vary sinusoidally with time:

$$\rho_v(\mathbf{R}_i, t) = \rho_v(\mathbf{R}_i)\cos(\omega t + \phi)$$

we can use phasor notation:

$$\rho_v(\mathbf{R}_i, t) = \mathfrak{Re}\left[\tilde{\rho}_v(\mathbf{R}_i)\, e^{j\omega t}\right],$$

with

$$\tilde{\rho}_v(\mathbf{R}_i) = \rho_v(\mathbf{R}_i)\, e^{j\phi}.$$

Expressions for potentials become:

$$\widetilde{V}(\mathbf{R}) = \frac{1}{4\pi\varepsilon}\int_{\mathcal{V}'} \frac{\tilde{\rho}_v(\mathbf{R}_i)\, e^{-jkR'}}{R'}\, d\mathcal{V}' \quad \text{(V)}.$$

$$\widetilde{\mathbf{A}}(\mathbf{R}) = \frac{\mu}{4\pi}\int_{\mathcal{V}'} \frac{\widetilde{\mathbf{J}}(\mathbf{R}_i)\, e^{-jkR'}}{R'}\, d\mathcal{V}',$$

Also:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \qquad \text{(dynamic case)}.$$

$$\widetilde{\mathbf{H}} = \frac{1}{\mu}\nabla \times \widetilde{\mathbf{A}}.$$

Maxwell's equations become:

$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}}$$

$$\text{or} \qquad \widetilde{\mathbf{H}} = -\frac{1}{j\omega\mu}\nabla \times \widetilde{\mathbf{E}}.$$

$$\nabla \times \widetilde{\mathbf{H}} = j\omega\varepsilon\widetilde{\mathbf{E}} \quad \text{or} \quad \widetilde{\mathbf{E}} = \frac{1}{j\omega\varepsilon}\nabla \times \widetilde{\mathbf{H}}.$$

$$k = \frac{\omega}{u_p}$$

What about <u>free-space</u> ???

$\mu \vec{H}$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\epsilon \vec{E}$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \times \mathbf{H} = \cancel{\mathbf{J}} + \frac{\partial \mathbf{D}}{\partial t}$$

0

$$\Downarrow$$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\nabla \times \vec{H}\right)$$

$$\Downarrow$$

$$\nabla \times \nabla \times \vec{E} = -\mu\epsilon \frac{\partial^2}{\partial t^2} \vec{E}$$

It's a <u>wave</u> eqtn.!

See ya' in ch. 7